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RESEARCH ON SIGNAL COMPENSATION PROCESSES IN THERMAL INDUSTRIAL OBJECTS WITH DISTRIBUTED PARAMETERS

The article examines a thermal industrial object with distributed temperature parameters. An information-geometric model based on transfer functions has been developed for the industrial object and a mathematical model has been determined. Additionally, a cross-coupling compensator has been synthesized. Sequential logarithmic identification method was used and studied to formulate the mathematical model of the object. Transmission coefficients, transport delay and time constants for each transfer function of the information-geometric model were calculated during this process. A compensator for three temperature channels was synthesized using matrix calculus approaches from linear algebra. Given the computational complexity of calculations, the authors developed and utilized specialized software capable of automatically real-time identification of smooth aperiodic transient responses and automatically synthesizing a cross-coupling compensator in real-time The Einstein digital temperature converter set was used for this purpose. It was assembled into a measurement channel with a digital output and a USB interface. The authors also investigated temperature stabilization channels in the studied industrial object using a PID controller and a relay actuator. A mathematical study of the amplitude and period of self-oscillations across the three control channels was conducted determining the impact of transport delay on the amplitude and period of temperature signal oscillations. Future research is planned for different types of industrial objects characterized by different transfer functions, specifically those involving high-frequency vibration processes. Vibration and its derivatives, namely vibrational displacement, velocity, and acceleration along with temperature, are among the most commonly monitored parameters in industrial applications..

Key words: industrial technological object, mathematical model, cross-coupling compensator, practical identification, temperature stabilization, regulator, transfer function, transient response

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ДОСЛІДЖЕННЯ ПРОЦЕСІВ КОМПЕНСАЦІЇ СИГНАЛІВ У ТЕПЛОВИХ ПРОМИСЛОВИХ ОБ'ЄКТАХ ІЗ РОЗПОДІЛЕНИМИ ПАРАМЕТРАМИ

У статті обраний і досліджений тепловий промисловий об'єкт з розподіленими температурними параметрами. Для промислового об'єкту побудована інформаційно-геометрична модель, що базується на передавальних функціях, визначена математична модель, синтезований компенсатор перехресних зв'язків. Для формування математичної моделі об'єкту використаний і досліджений метод послідовного логарифмування практичної ідентифікації, у ході якого були розраховані коефіцієнти передач, транспортне запізнення та сталі часу кожної передавальної функції інформаційно-геометричної моделі. Використовуючи підходи матричного числення лінійної алгебри був синтезований компенсатор для трьох температурних каналів. Через велику надмірність розрахунків авторами створене і використане спеціалізоване програмне забезпечення, яке спроможне автоматично проводити практичну ідентифікацію гладких аперіодичних перехідних характеристик в тестовому он-лайн режимі і автоматично синтезувати компенсатор перехресних зв'язків у режимі реального часу. Для цього користувався комплект цифрового перетворювача температури Einstein, який зібраний у вимірювальний канал з цифровим виходом і USB вихідним портом Авторами досліджені канали стабілізації температури в досліджуваному промисловому об'єкті з ПІД-регулятором і релейним виконавчим елементом. Здійснене математичне дослідження амплітуди і періоду автоколивань по трьом каналам управління, визначений вплив транспортного запізнення на значення амплітуди і періоду автоколивань температурних сигналів. Подальші дослідження заплановано провести з іншим типом промислових об'єктів які характеризуються іншими передавальними функціями, а саме з вібраційними високочастотними процесами. Вібрація і її похідні – вібропереміщення, виброшвидкість, віброприскорення, як і температура – є найбільш вживаними параметрами в промисловості.

Ключові слова: промисловий технологічний об'єкт, математична модель, компенсатор перехресних зв'язків, практична ідентифікація, стабілізація температури, регулятор, передавальна функція, перехідна характеристика

Introduction. Modern industrial automation tools play a crucial role in enabling these objects to carry out technological processes. Precise control in such systems is essential for achieving key performance parameters such as minimal regulation error and high responsiveness [1, 2].

The development of control systems based on the main synthesis algorithm includes the following steps [3, 4]:

- selecting the object of study and analyzing its parameters;
- constructing an information model of the object, where the object is generally represented as a «black box»;
- classifying all signals affecting the object according to the principle: observable and controllable; observable and uncontrollable; unobservable and uncontrollable;
- identifying the primary physical parameters from the observable and controllable signals for mathematical analysis and achieving the target function in control;
- performing simulation modeling;
- creating an experimental prototype of the control system.

Another important aspect in developing industrial control systems is choosing an appropriate mathematical model structure [5]:

- object with lumped parameters;
- object with distributed parameters.

Since all industrial technological systems exhibit distributed parameters, while control system theory is based on working with objects with lumped parameters, there is a practical need to represent the studied object as an object with lumped parameters while highlighting cross-coupling [6]. The identification and compensation of cross-coupling influences are crucial tasks for achieving the highest control performance.

As the object of study, the authors selected an object from the series of multi-zone tunnel-type systems [7], where the distribution of both physical and geometric parameters is clearly evident. The selected object [7] is shown in Fig. 1.

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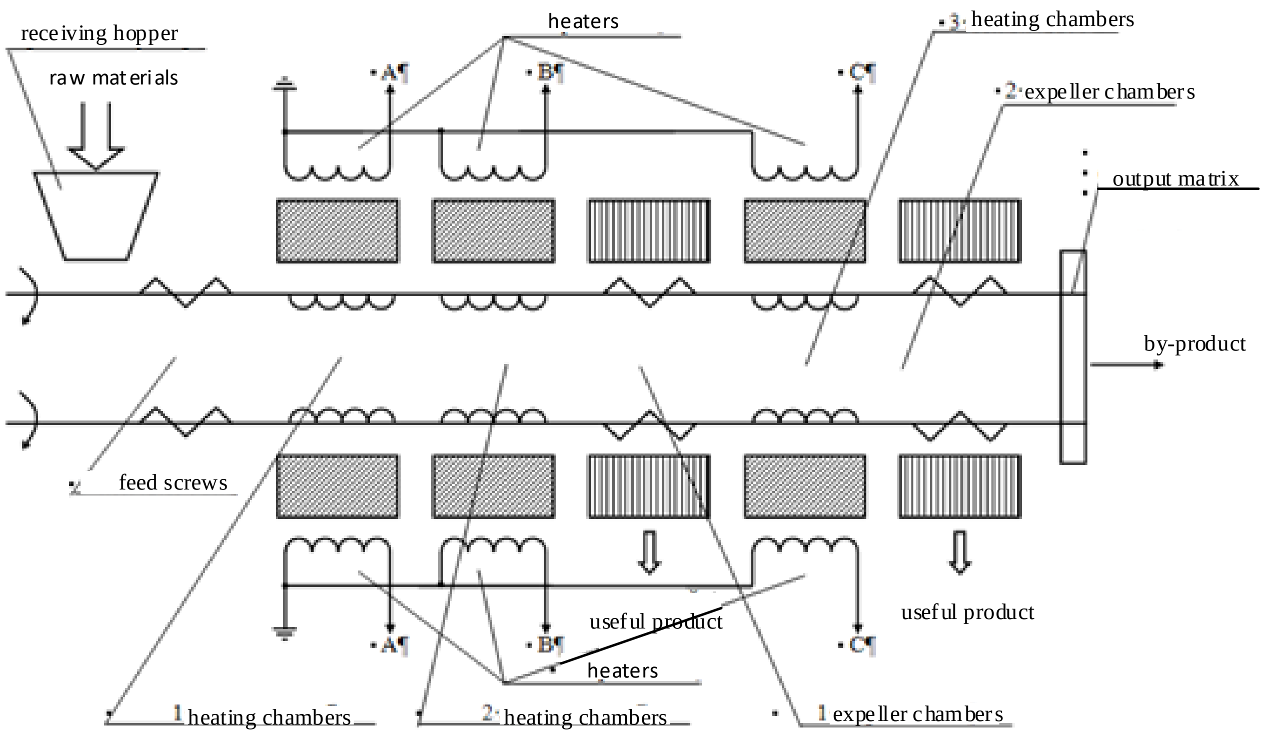

Fig. 1 – Multi-zone tunnel-type technological unit

The industrial technological object [7, 8] shown in Fig. 1 is designed for obtaining vegetable oils from oilseed agricultural crops consisting of:

- a receiving hopper;
- feed screws;
- three heating chambers;
- two expeller chambers;
- an output matrix.

The quality of the final product from such an industrial technological object directly depends on the stability of maintaining the set temperatures in the heating chambers [7].

In [9], an information-geometric model of the object shown in Fig. 1 is presented and depicted in Fig. 2.

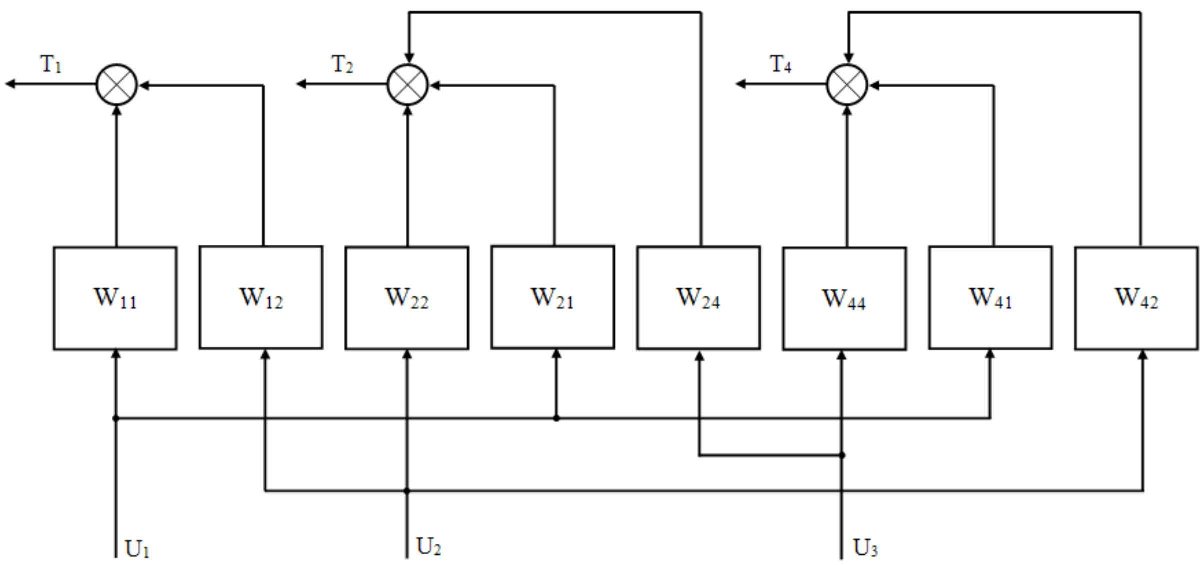

Fig. 2 – information-geometric model of the object:

U – input signals; *T* – output signals; W_{ii}, W_{ij} – transfer functions

The information-geometric model of the object in Fig. 2 is presented in the form of a schematic representation with transfer functions W_{ii}, W_{ij} and is obtained through the discretization of control disturbances.

In [7, 9], a mathematical model of the considered object is obtained through practical identification:

$$W_O = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix} \quad (1)$$

in the form of:

$$W_O = \begin{bmatrix} \frac{44.6e^{-0.4p}}{(1+5.9p)(1+2.9p)} & \frac{6.9e^{-6.5p}}{(1+9.5p)(1+2.0p)} & 0 \\ \frac{6.3e^{-5.3p}}{(1+12.6p)(1+4.0p)} & \frac{64.2e^{-0.2}}{(1+3.2p)(1+1.7p)} & \frac{1.0e^{-21.0p}}{(1+14.1p)(1+4.4p)} \\ \frac{5e^{-12.1}}{(1+11.5p)(1+3.6p)} & \frac{6.1e^{-7.8p}}{(1+9.7p)(1+6.8p)} & \frac{45.2e^{-0.1p}}{(1+2.6p)(1+3.4p)} \end{bmatrix}. \quad (2)$$

In models (1) and (2):

W_{ii} represents the transfer functions of direct influences (useful signals);

W_{ij} represents the transfer functions of cross influences (undesirable signals).

To construct a highly accurate control system for the studied object, it is necessary to eliminate the W_{ij} elements, which can be practically achieved through compensator synthesis.

Research objective and problem statement. The primary objective of this study is the synthesis of a cross-coupling compensator in the thermal information-geometric model of an industrial multi-zone tunnel-type technological unit.

To achieve this objective, the following tasks must be completed:

- study the thermal information-geometric model of the investigated object;
- evaluate the validity of using practical identification methods to obtain the mathematical model of the studied object;
- select a scheme for incorporating the cross-link compensator;
- develop an algorithm for the synthesis of the compensator for an object with distributed temperature parameters;
- calculate and analyze the cross-coupling compensator;
- assess the accuracy of temperature stabilization by integrating a PID controller into the i^{th} channel.

Conducting research on selecting practical identification method. As the practical identification method, the study [7] selected the sequential logarithmization method. The sequential logarithmization method is used for identifying smooth non-oscillatory transient characteristics, represented by the following expression [10]:

$$h(t) \approx c_0 - \sum_{i=1}^{n} c_i e^{-\alpha_i t},$$

where $c_0 = h_\infty \approx h(T_y)$, c_i and α_i – real numbers, where the roots of the characteristic equation α_i must satisfy the empirical inequality.

$$\frac{\alpha_i}{\alpha_{i+1}} \le 0.5 \div 0.7\,;\; i = 1, 2, ..., n-1\,.$$

These conditions imply that the transfer function $W(p)$ has only simple poles, which are located at a sufficiently large distance from each other along the real axis.

The core idea of the method is the sequential approximation of transient characteristics $h(t)$, initially by solving a first-order equation, i.e., using the function $c_1 e^{-\alpha_1 t}$. If this approximation is unsatisfactory, a second component $c_2 e^{-\alpha_2 t}$ is introduced, meaning the order of the approximating equation is taken as 2, and so on. The unknown parameters c_i and α_i are determined at each stage of the approximation through the logarithmization operation, which is why this method is called the sequential logarithmization method. The sequence of actions for its application is as follows:

1. An experiment is conducted to obtain the transient characteristic of the thermal object.
2. The discretization time Δt is determined.

3. A tabular dependency $T_i^\circ = f\left(t_i\right)$ is constructed.

4. A graphical dependency $T^\circ = f\left(t\right)$ is constructed.

5. The pure delay time τ is truncated.

6. The pure delay time k. $k = \frac{T_{уст}^\circ}{A_{вх}}$ is truncated.

7. A tabular dependence $\left|h_1\right|_i = f\left(t_i\right)$ $\left|h_1\right|_i = T_{уст}^\circ - T_i^\circ$ is constructed.

8. A tabular dependence $lg\left|h_1\right|_i = f\left(t_i\right)$ is constructed.

9. A graphical dependence $lg\left|h_1\right| = f\left(t\right)$ is constructed.

10. An asymptote to the dependence $lg\left|h_1\right| = f\left(t\right)$ is performed when $t \to \infty$.

11. The parameters $lg\, c_1$ and t_1 are determined.

12. The parameter α_1. $\alpha_1 = \frac{lg\, c_1}{t_1}$ is determined.

13. The time constant T_1. $T_1 = \frac{1}{\alpha_1}$ is determined.

14. A tabular dependence $c_i = c_1 e^{-\alpha_1 t_i}$ is constructed.

15. A tabular dependence $\left|h_2\right|_i = \left|h_1\right|_i - c_1 e^{-\alpha_1 t_i}$ is constructed.

16. The numerical values of the residual functions $\left|h_2\right|_i$ are evaluated, after which the order of the transfer function $W\left(p\right)$ is determined.

17. Steps 8–16 of this algorithm are repeated.

18. An iterative process is carried out until the residual functions $\left|h_j\right|_i \approx 0$ satisfy the required conditions across the entire time range.

19. The transfer function (mathematical model) is expressed as [7]:

$$W\left(p\right) = \frac{k}{\left(1 + pT_1\right)\left(1 + pT_2\right)...\left(1 + pT_n\right)} e^{-p\tau}.$$

The algorithm is graphically represented in Fig. 3.

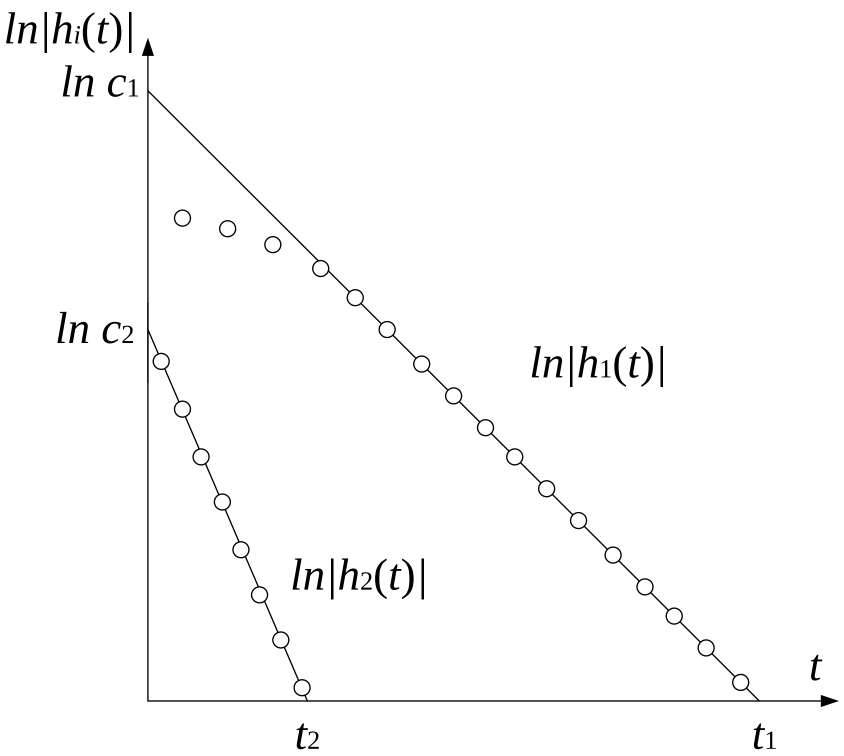

Fig. 3 – Graphical interpretation of the sequential logarithmization method.

The practical application of the sequential logarithmization method for determining dynamic characteristics based on the transient responses of industrial objects shows that $h(t)$ can be approximated by a sum of two to four exponentials [11]. It should be noted that the determination of the coefficients c_i and roots α_i is carried out based on the transient response, from which the pure delay time τ has already been extracted and the transfer coefficient k has been determined.

The authors, under laboratory conditions, have demonstrated the feasibility of using the sequential logarithmization method [12] (Fig. 4) to obtain a mathematical model in the form of:

$$W(p) = \frac{k}{\prod_{i=1}^{n} (T_i p + 1)} e^{-p\tau}, \quad (3)$$

where

k – transfer coefficient;

T_i – time constants;

τ – transport delay.

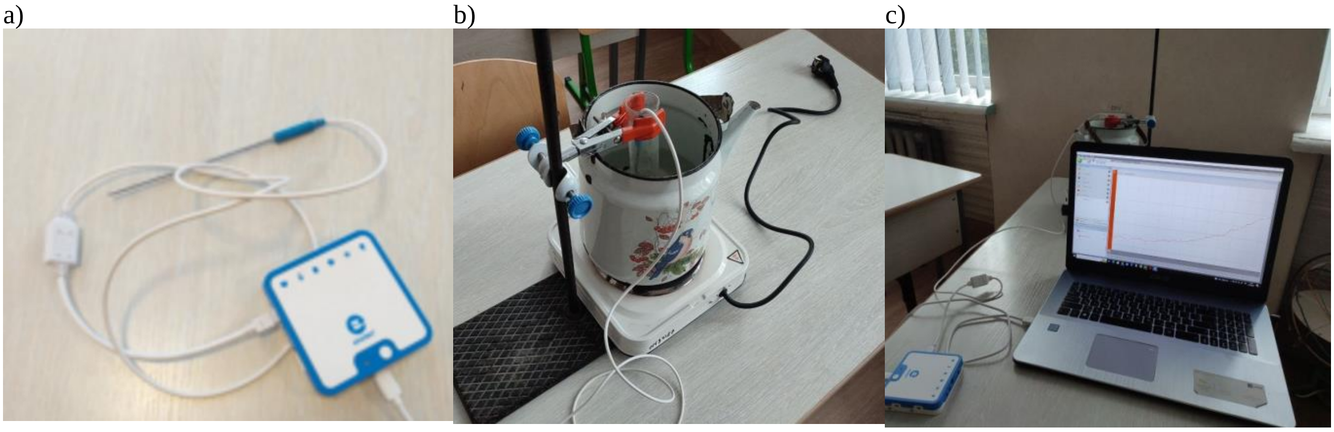

Fig. 4 – Application of the sequential logarithmization method in laboratory conditions:
a) temperature sensor with a copper-sensitive element;
b) laboratory setup of the thermal object;
c) Setup for recording transient characteristics

Synthesis of cross-coupling compensator. To compensate for cross-coupling, a compensator model needs to be introduced into the model of the studied object [13]. The compensator is integrated sequentially into the transfer function of the object. Essentially, two approaches to implementing the compensator can be considered [14] (Fig. 5).

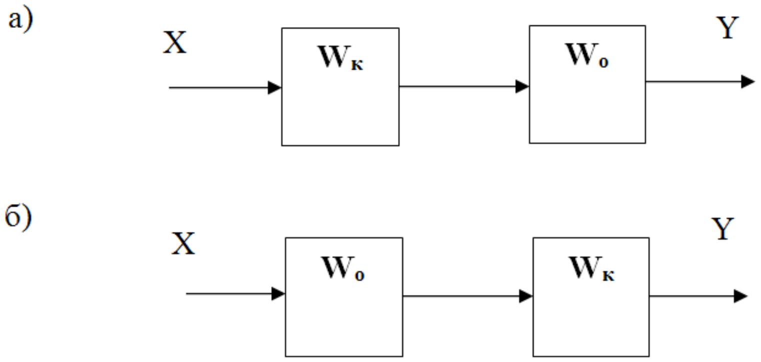

Fig. 5 – Compensator connection schemes

The first compensator implementation scheme (Fig. 5a) is designed to decouple the channels of any multi-connected system using a local controller and is typically applied in closed-loop systems. The second scheme (Fig. 5b) compensates for cross influences by automatically introducing appropriate corrections and is used in open-loop systems [14].

Both schemes have advantages and disadvantages. Since the scheme in Fig. 5b is more versatile, it is recommended as the primary approach.

Introducing a compensator using the scheme in Fig. 5b for channel decoupling should transform the studied object from the form (1) into the following form:

$$W(p)=\begin{bmatrix} W_{11} & 0 & 0 \\ 0 & W_{22} & 0 \\ 0 & 0 & W_{33} \end{bmatrix}, \qquad (4)$$

when only the main diagonal elements of the matrix remain in the model, while all off-diagonal elements are reduced to zero

to achieve such compensation, it is advisable to use the algorithm from [15], according to which:

$$W_K(p)=\begin{bmatrix} \frac{b_{11}}{\Delta} & \frac{b_{12}}{\Delta} & \frac{b_{13}}{\Delta} \\ \frac{b_{21}}{\Delta} & \frac{b_{22}}{\Delta} & \frac{b_{23}}{\Delta} \\ \frac{b_{31}}{\Delta} & \frac{b_{32}}{\Delta} & \frac{b_{33}}{\Delta} \end{bmatrix} \cdot \begin{bmatrix} W_{11} & 0 & 0 \\ 0 & W_{22} & 0 \\ 0 & 0 & W_{33} \end{bmatrix} \cdot . \qquad (5)$$

B (5):

- b_{ij} – algebraic complements of the elements $[W_0]_{ji}$ of the model (1);
- Δ – determinant of the model (1).

The implementation procedure of (5), according to [13], is defined through equations for open-loop systems

$$W_0(p)\ W_\kappa(p) = \text{diag}W_0(p)=W_\Sigma(p)$$

Having the following form:

$$W_\kappa(p)=W_0^{-1}(p)\ \text{diag}W_0(p), \qquad (6)$$

where $W_\kappa(p)$, $W_0(p)$, diag$W_0(p)$ – dynamic models of the compensator, the object without compensation and the object with cross-coupling compensation, respectively

Based on (6), the compensator model takes the following form

$$W_\kappa(p)=\begin{bmatrix} \frac{b_{11}W_{11}}{\Delta} & \frac{b_{12}W_{22}}{\Delta} & \frac{b_{13}W_{33}}{\Delta} \\ \frac{b_{21}W_{11}}{\Delta} & \frac{b_{22}W_{22}}{\Delta} & \frac{b_{23}W_{33}}{\Delta} \\ \frac{b_{31}W_{11}}{\Delta} & \frac{b_{32}W_{22}}{\Delta} & \frac{b_{33}W_{33}}{\Delta} \end{bmatrix}, \qquad (7)$$

And the compensation model for the channels, respectively, is as follows:

$$W_{\kappa 1}(p)=\begin{bmatrix} \frac{b_{11}}{\Delta}W_{11} \\ \frac{b_{12}}{\Delta}W_{22} \\ \frac{b_{33}}{\Delta}W_{33} \end{bmatrix},\ W_{\kappa 2}(p)=\begin{bmatrix} \frac{b_{21}}{\Delta}W_{11} \\ \frac{b_{22}}{\Delta}W_{22} \\ \frac{b_{23}}{\Delta}W_{33} \end{bmatrix},\ W_{\kappa 3}(p)=\begin{bmatrix} \frac{b_{31}}{\Delta}W_{11} \\ \frac{b_{32}}{\Delta}W_{22} \\ \frac{b_{33}}{\Delta}W_{33} \end{bmatrix}. \qquad (8)$$

For the original model (2) of the studied object, equation (5) takes the following form:

$$W_K=\begin{bmatrix} \frac{b_{11}}{\Delta} & \frac{b_{12}}{\Delta} & \frac{b_{13}}{\Delta} \\ \frac{b_{21}}{\Delta} & \frac{b_{22}}{\Delta} & \frac{b_{23}}{\Delta} \\ \frac{b_{31}}{\Delta} & \frac{b_{32}}{\Delta} & \frac{b_{33}}{\Delta} \end{bmatrix} \cdot \begin{bmatrix} \frac{44.6\cdot e^{-24p}}{(147p+1)(354p+1)} & 0 & 0 \\ 0 & \frac{64.2\cdot e^{-12p}}{(192p+1)(102p+1)} & 0 \\ 0 & 0 & \frac{45.6\cdot e^{-6p}}{(156p+1)(204p+1)} \end{bmatrix}. \qquad (9)$$

The results of calculating the elements of the left matrix in (9) are summarized in Table 1.

Table 1 – The results of calculating the elements of the left matrix in (9)

Element	Calculation results
$\frac{b_{11}}{\Delta}$	$-50e^{10.8p}(2p+1)\cdots(63p+5)\left(4.16\cdot 10^{6}p-1.81\cdot 10^{8}e^{28.5p}-6.35\cdot 10^{9}pe^{28.5p}+\cdots+3.81\cdot 10^{5}\right)/\Delta$
$\frac{b_{12}}{\Delta}$	$-15590e^{33.0p}(4p+1)\cdots(63p+5)/\Delta$
$\frac{b_{13}}{\Delta}$	$690e^{12.1p}(4p+1)\cdots(63p+5)/\Delta$
$\frac{b_{21}}{\Delta}$	$5e^{6.5p}(2p+1)\cdots(59p+10)\left(1.41\cdot 10^{6}p-3.56\cdot 10^{6}e^{27.7p}-\cdots+62500\right)/\Delta$
$\frac{b_{22}}{\Delta}$	$5.04\cdot 10^{6}e^{39.1p}(2p+1)\cdots(63p+5)/\Delta$
$\frac{b_{23}}{\Delta}$	$-2.23\cdot 10^{5}e^{18.2p}(2p+1)\cdots(63p+5)/\Delta$
$\frac{b_{31}}{\Delta}$	$7.5e^{26.5p}(2p+1)\cdots(59p+10)\left(1.28\cdot 10^{7}p-5.35\cdot 10^{6}e^{0.8p}-\cdots+6.41\cdot 10^{5}\right)/\Delta$
$\frac{b_{32}}{\Delta}$	$10e^{21p}(4p+1)\cdots(63p+5)\left(8.73\cdot 10^{6}p-2.72\cdot 10^{6}e^{10.4p}-\cdots+3.45\cdot 10^{5}\right)/\Delta$
$\frac{b_{33}}{\Delta}$	$-0.3e^{27.8p}(17p+5)\cdots(97p+10)(13p+5)\left(9.93\cdot 10^{7}p-4.77\cdot 10^{8}e^{11.2p}-\cdots+7.25\cdot 10^{6}\right)/\Delta$
Δ	$5.69\cdot 10^{13}p+4.04\cdot 10^{15}e^{38.9p}-8.5\cdot 10^{12}e^{10.4p}-\cdots+1.08\cdot 10^{12}$

Table 2 presents the elements of the compensator according to expression (7).

Table 2 – The results of calculating the compensator elements according to (7)

Element	Calculation results
$\frac{b_{11}W_{11}}{\Delta}$	$-5.965\cdot 10^{8}e^{15.5p}(2p+1)\cdots(63p+5)\left(2.535\cdot 10^{8}p-8.198\cdot 10^{10}e^{36.2p}-\cdots+2.326\cdot 10^{7}\right)/\Delta$
$\frac{b_{12}W_{22}}{\Delta}$	$-7.99\cdot 10^{12}e^{37.8p}(4p+1)\cdots(63p+5)/\Delta$
$\frac{b_{13}W_{33}}{\Delta}$	$7.822\cdot 10^{9}e^{17p}(4p+1)\cdots(63p+5)/\Delta$
$\frac{b_{21}W_{11}}{\Delta}$	$2.164\cdot 10^{8}e^{13.9p}(2p+1)\cdots(59p+10)\left(1.766\cdot 10^{7}p-4.022\cdot 10^{8}e^{39.7p}-\cdots+7.812\cdot 10^{5}\right)/\Delta$
$\frac{b_{22}W_{22}}{\Delta}$	$1.956\cdot 10^{16}e^{51.7p}(2p+1)\cdots(63p+5)/\Delta$
$\frac{b_{23}W_{33}}{\Delta}$	$-1.915\cdot 10^{13}e^{30.9p}(2p+1)\cdots(63p+5)/\Delta$
$\frac{b_{31}W_{11}}{\Delta}$	$-1.291\cdot 10^{9}e^{34.7p}(2p+1)\cdots(59p+10)\left(8.854\cdot 10^{9}p-3.907\cdot 10^{7}e^{3.5p}-\cdots+2.675\cdot 10^{8}\right)/\Delta$
$\frac{b_{32}W_{22}}{\Delta}$	$3.895\cdot 10^{8}e^{20.8p}(4p+1)\cdots(63p+5)\left(1.833\cdot 10^{8}p-5.822\cdot 10^{8}e^{15.5p}-\cdots+7.245\cdot 10^{6}\right)/\Delta$
$\frac{b_{33}W_{33}}{\Delta}$	$-38310.0e^{39.7p}(34p+5)\cdots(97p+10)\left(1.271\cdot 10^{11}p-5.106\cdot 10^{12}e^{12p}-\cdots+9.281\cdot 10^{9}\right)/\Delta$
Δ	$9.115\cdot 10^{17}p-8.88\cdot 10^{18}e^{39.7p}+4.89\cdot 10^{21}e^{51.7p}-\cdots+1.726\cdot 10^{16}$

The channel compensation models, based on (8), are formed from Table 2. The determination of real compensation models, according to (2), using expressions (5), (7), and (8), was carried out with the help of software, the fragments of which are presented below.

The description of the transfer function (2):

```
  function w = generate_transfer_function(obj, k, tau, time1, time2)
    % Generate the transfer function according to the rule:
    % w = (k * exp(-tau*p)) / ((1 + time1*p) * (1 + time2*p))
    w = simplify((k * exp(-tau * obj.p)) / ((1 + time1 * obj.p) * (1 + time2 * obj.p)));
 end
```

The calculation of the inverse matrix (6):

```
function W_inv = compute_inverse_matrix(obj, numerical)
        % Compute the inverse matrix (symbolic or numeric)
        W_matrix = obj.functions;
        try
           det_val = double(subs(det(W_matrix), obj.p, 1));
            singular = abs(det_val) < 1e-8;
        catch
           singular = false; % fallback
       end

          if singular
              W_inv = pinv(W_matrix);
          else
              W_inv = inv(W_matrix);
          End

          % Apply symbolic simplification for Laplace context
          W_inv = arrayfun(@(x) BaseType.advanced_simplify_laplace(x, obj.p), W_inv);
        end
    end
```

The calculation of the compensator matrix (7):

```
function W_k = compute_compensator(obj)
        % Compute compensator: W_k(p) = diag(W_0(p)) * W_0^-1(p)
        W_matrix = obj.functions;
        W_diag = diag(diag(W_matrix));
        W_inv = obj.compute_inverse_matrix(false);  % Use symbolic inversion
        W_k = W_diag * W_inv;
    end
```

Analytical studies of temperature signals in temperature control systems. Full or partial compensation for the influence of cross-coupling effects on the considered control channel is insufficient for stabilizing the temperature in the corresponding heating chamber. Given that a relay actuator is used for temperature control in any industrial technological object, it is advisable to choose a PID controller as the temperature stabilization regulator, which includes proportional, integral, and derivative components [16]. Such a controller is characterized by the presence of self-oscillations in the steady-state.

When calculating the amplitude and period of the self-oscillations of the temperature signals in the control channels with a PID controller, the transition function is first considered without accounting for transport delay [17]. For this case, the algebraic equation $T=W(p)U$ is implemented with the corresponding transfer function:

$$T_{\tau=0}(p)=\frac{U}{p}\frac{k}{(T_1 p+1)(T_2 p+1)}\,. \qquad (9)$$

The original image (9), according to the inverse Laplace transform [19]:

$$T_{\tau=0}(t)=kU\left(1+\frac{T_1 e^{-t/T_1}-T_2 e^{-t/T_2}}{T_2-T_1}\right)+C\,, \qquad (10)$$

where C – the integration constant.

When $\tau=t$ із (10), the expression is following:

$$T_{\tau=0}(t)=kU\left(1+\frac{T_1\,e^{-\tau/T_1}-T_2\,e^{-\tau/T_2}}{T_2-T_1}\right)+C=-\Delta\,. \tag{11}$$

From (11), the integration constant is:

$$C=-\Delta-kU\left(1+\frac{T_1\,e^{-\tau/T_1}-T_2\,e^{-\tau/T}}{T_2-T_1}\right). \tag{12}$$

Taking into account (12), equation (10) takes the form:

$$T_{\tau=0}(t)=kU(1+\frac{T_1\,e^{-t/T_1}-T_2\,e^{-t/T_2}}{T_2-T_1})-\Delta-kU(1+\frac{T_1\,e^{-\tau/T_1}-T_2\,e^{-\tau/T_2}}{T_2-T_1}) \tag{13}$$

For $t=t_1$, when the temperature oscillation T of the channel reaches its maximum, from (13) we obtain:

$$2\Delta=kU\left(1+\frac{T_1\,e^{-t_1/T}-T_2\,e^{-t_1/T_2}}{T_2-T_1}\right)-kU\left(1+\frac{T_1\,e^{-\tau/T_1}-T_{B2}\,e^{-\tau/T_2}}{T_2-T_1}\right)$$

Further transformations allow us to obtain t_1 [9]:

$$t_1=\sqrt{2T_1T_2\left(\frac{2\Delta}{kU}+\frac{T_1\,e^{-\tau/T_1}-T_2\,e^{-\tau/T_2}}{T_2-T_1}+1\right)}\,. \tag{14}$$

Increasing t_1 in (14) by τ, we obtain the half-period of oscillation. Then, the period Q is determined as:

$$Q=2(t_1+\tau)=2\left[\tau+\sqrt{2T_1T_2\left(\frac{2\Delta}{kU}+\frac{T_1\,e^{-\tau/T_1}-T_2\,e^{-\tau/T_2}}{T_2-T_1}+1\right)}\right]. \tag{15}$$

The amplitude of the oscillations A is determined from (13) at t=0:

$$A=\Delta+kU\left(1+\frac{T_1\,e^{-\tau/T_1}-T_2\,e^{-\tau/T_2}}{T_2-T_1}\right).$$

The numerical values of the amplitudes and periods of the temperature oscillations in the main channels with the specified parameters Δ are presented in Table 3.

Table 3 – The values of the amplitudes and periods of temperature oscillations

	A,°C	Q, min
W_{11}	2.19	4.39
W_{22}	2.22	2.09
W_{33}	2.03	2.71

The determination of the amplitudes and periods of temperature oscillations (Table 3) was carried out using MathLab 7.11 software. The program fragments are provided below.

The determination of the oscillations amplitude:

```
function A = compute_amplitude(~, varargin)
  % Compute amplitude of oscillations
  k = varargin{1};
  T1 = varargin{2};
  T2 = varargin{3};
  tau = varargin{4};
  delta = varargin{5};
  U = varargin{6};
```

```
  A = delta + k*U * (1 + (T1 * exp(-tau / T1) - T2 * exp(-tau / T2)) / (T2 - T1));
  end
```

The determination of the oscillation period:

```
function Q = compute_period(~, varargin)
  % Compute period of oscillations using given formula
  tau = varargin{1};
  k = varargin{2};
  T1 = varargin{3};
  T2 = varargin{4};
  delta = varargin{5};
  U = varargin{6};

  Q = 2*(tau + sqrt(2*T1*T2 * ((2*delta/k*U) + (T1 * exp(-tau/T1) - T2*exp(-tau/T2)) / (T2 - T1) + 1)));
end
```

Conclusion. During the research, a cross-coupling compensator was synthesized and the stabilization process of temperature signals in the heating chambers of a multi-zone continuous technological unit was analyzed.

In the course of the study:

- the research object was selected, characterized by a pronounced spatial distribution of temperature parameters.
- the functional scheme of the selected object was analyzed.
- an information-geometric model of the object was considered based on the use of transfer functions.
- the mathematical model of the selected object was identified.

The method of practical identification through sequential logarithmization was chosen as the basis to obtain the mathematical model. This method is best suited for smooth aperiodic transient characteristics without overshoot. The authors refined this method under laboratory conditions. For the experiments, the Einstein primary transducer (sensor) was used. It is integrated into a measurement channel with a digital output and a USB port. The software interface, which queries the port using the appropriate communication protocol, is provided by the manufacturer.

The cross-coupling compensator was synthesized using a well-known algorithm and is based on transforming the distributed mathematical model of the studied industrial object into a lumped-parameter representation. The compensator synthesis employs linear algebraic operations on the mathematical model of the industrial object including the computation of algebraic complements, the determinant and a diagonal matrix.

The synthesis process involves handling large mathematical expressions. To efficiently perform these operations and optimize the resulting expressions, the authors developed and utilized specialized software.

To stabilize the obtained temperature channels and maintain the temperature at predefined levels, a PID controller with a relay actuator is used. Such approaches in temperature stabilization lead to self-oscillatory processes. The authors have studied these self-oscillatory processes, analyzed the contributions of the gain coefficient, time constants, and transport delay, and analytically derived the amplitude and period of these oscillations.

Future research should generalize approaches to the design of cross-coupling compensators, address challenges related to transfer functions of different types (e.g., periodic oscillatory functions) and tackle the problem of automatic mathematical modeling of industrial technological objects and compensators using modern computational techniques.

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